Optimal Power Allocation for Energy Recycling Assisted Cooperative Communications

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Abstract—We investigate the problem of optimal power allocation for energy recycling cooperative communications systems, employing full duplex relays, based on the criterion of maximizing the rate, or equivalently the Signal to Noise Ratio (SNR), of the system. A system model is investigated where each time slot is split into an information transmission phase, during which the Source (S) transmits information to the destination (D) and a full-duplex Relay (R), and an energy harvesting phase. During the energy harvesting phase, R relays information to D, while concurrently it performs energy harvesting, exploiting a signal transmitted by S and energy recycling, exploiting its own transmission. For this system model, we formulate a rate/SNR maximization problem, in order to compute the optimal source power levels for both information transfer and energy transfer phases. The cost function of this optimization problem is then substituted by a sharp approximation, which allows for obtaining an analytically tractable power allocation. The performance of the resulting power allocation is then assessed by means of Monte Carlo simulations, and it is found that it outperforms existing solutions. It is therefore shown that our proposed solution can contribute towards increasing the range of IoT networks.

Index Terms—Energy harvesting, energy recycling, cooperative communications.

I. INTRODUCTION

Internet of things (IoT) is expected to be a prominent feature of 5G wireless communications. While the massive deployment of sensors, fundamental to the idea of IoT, promises great economic and social impact, the long-term sustainability of such deployments must be addressed. To this end, Energy Harvesting (EH) and Wireless Power Transfer (WPT) have been considered as candidate solutions for powering IoT networks. An important flavor of WPT systems are the Simultaneous Wireless Information and Power Transfer (SWIPT) systems that employ time sharing (TS) or power splitting (PS) methods, [1], to enable the transfer of both data and energy to users. In TS, a portion of the time slot is dedicated for energy harvesting while the rest of the time slot is used for information transfer. On the other hand, PS allows extraction of both power and information at the same time, but the signal power is split into two parts, one for energy harvesting, and one for information transfer. While initially the TS and PS solutions were proposed for half duplex communications, their application in full duplex communications has also been investigated and has been found to result in significant improvements in throughput over half duplex systems [2].

The use of full duplex solutions in cooperative communications systems is considered in several works [3]–[7]. Within the context of WPT systems, full duplex solutions can be exploited in order to combine data transmission using some of the available antennas of the wireless transmitter, with energy harvesting, using the remaining antennas of the radio equipment. Such a solution can allow the transmitter to harvest part of the energy that it transmits, introducing the concept of energy recycling. Recently, energy recycling has found several applications in cooperative communications systems. In more detail, in [4] the authors discuss an energy recycling relay system with the aim of maximizing throughput. Reference [8] generalizes the work in [4] focusing on a Multiple Input Multiple Output (MIMO) relay system. The authors provide optimal power allocation strategies for downlink and joint uplink/downlink cases. Moreover, in [9], the authors study the beamforming optimization problem to maximize the achievable rate for an energy recycling cooperative communications system, subject to a constraint on the available transmitted power at the relay node. Similarly, in [10] a decode and forward full duplex relay system with SWIPT is considered. The fundamental trade-off between end to end signal to interference plus noise ratio and the recycled power is studied.

Motivated by the above, in this work we investigate further applications of energy recycling in cooperative communications systems. In more detail, we tackle the problem of optimal power allocation for cooperative, energy recycling networks, that has not been investigated in the technical literature, focusing on the criterion of maximizing the instantaneous communication rate, or equivalently, maximizing the Signal to Noise Ratio (SNR) of the communication channel. To this end, we introduce an approximation for the SNR and, based on this approximation, we present a method for maximizing the SNR at the receiver.

The paper is structured as follows. In Section II, we present the considered system model, while in Section III we present a technique for approximating the SNR of the system, and design a power allocation scheme such as to maximize this SNR approximation. The proposed solution is derived by solving a quartic equation. Hence, it can be easily computed using standard algebraic techniques. Section IV presents numerical results that prove the validity of the derived SNR approximation, and illustrate the gains of the

proposed power allocation scheme. Finally, in Section V, we present our conclusions.

Notation: We use lower case bold letters to denote vectors, and upper case bold letters to denote matrices. Notation $\mathbf{A}[k,l]$ is used to denote the element of row k and column l of matrix $\mathbf{A}$. $\mathbf{A}^T$ stands for the transpose of matrix $\mathbf{A}$. We use notation $x \sim \mathcal{CN}\left(0, \sigma^2\right)$ to indicate that random variable x follows a complex Gaussian distribution with mean value equal to zero and variance σ^2. Operator $(\cdot)^{\star}$ is used to denote complex conjugation, and operator $*$ stands for the linear, discrete convolution.

II. System model

A cooperative system comprising a source (S) with a single transmit antenna is considered that communicates with a destination (D) equipped with one receive antenna. Communication is achieved by employing a full-duplex relay R, equipped with two antennas, that is powered by means of energy harvesting. In more detail, R harvests energy transmitted by S, as well as energy transmitted by itself. Such a system can typically be found in indoor industrial IoT applications where S is connected with grid power, while the other sensors may not be connected with grid power and need to be charged by the RF signal, to act as relays for the source information signals. Without loss of generality, we assume that the S-D link is weak, necessitating the use of the S-R-D link in order to assist communication.

Focusing on the relay R, one of its antennas, antenna 1, is devoted to receiving data and energy from the source S, while antenna 2 is devoted to relaying information to D, employing an Amplify and Forward (AF) scheme. Moreover, D also receives information through the direct S-D link. In more detail, each time slot of the communications scheme, that is of duration of T seconds, is split in the two following phases, where each one has a duration equal to $T/2$.

A. Phase I: Data transmission from S

In the first phase, the source S transmits data, that are received by the relay. The signal received at relay R in this phase is expressed as

$$y_{r,1} = h_{sr}\sqrt{P_{s,1}}x_{s,1} + n_{r,1}, \tag{1}$$

where $P_{s,1}$ is the transmit power of S during the first phase of the time slot, h_{sr} characterizes the S-R channel, $x_{s,1}$ is the signal transmitted by S during this phase, and $n_{r,1} \sim \mathcal{CN}\left(0, \sigma_r^2\right)$, is the Additive White Gaussian Noise (AWGN) at R. Concurrently, D also receives a distorted replica of the signal transmitted by S. In more detail, the signal received by D is expressed as

$$y_{d,1} = h_{sd}\sqrt{P_{s,1}}x_{s,1} + n_{d,1}, \tag{2}$$

where h_{sd} characterizes the S-D channel, and $n_d \sim \mathcal{CN}\left(0, \sigma_d^2\right)$ is the AWGN at D. One can easily see that the SNR for the communication on this link is expressed as

$$\gamma_{sd} = \frac{g_{sd}P_{s,1}}{\sigma_d^2}, \text{ where } g_{sd} = |h_{sd}|^2. \tag{3}$$

B. Phase II: Data transmission/Energy Harvesting from/at R

In the second phase, the signal received by R during the first phase is amplified and forwarded to D using a power level P_r, while concurrently, another signal $x_{s,2}$ is transmitted by S, to be exploited by R, for energy harvesting purposes. The signal reaching D is then written as:

$$
\begin{aligned}
y_{d,2} &= \\
&= \frac{\sqrt{P_{s,1}P_r}}{\sqrt{A}}h_{sr}h_{rd}x_{s,1} + \frac{\sqrt{P_r}}{\sqrt{A}}h_{rd}n_{r,1} + \sqrt{P_{s,2}}h_{sd}x_{s,2} \\
&\quad + n_{d,2},
\end{aligned}
\tag{4}
$$

where $A = P_{s,1}|h_{sr}|^2 + \sigma_r^2$. $h_{r,d}$ denotes the R-D channel while $n_{d,2} \sim \mathcal{CN}\left(0, \sigma_d^2\right)$, $x_{s,2}$, is the signal transmitted by S so that R can perform energy harvesting, and $P_{s,2}$ is the power used by S for its transmission.

Focusing on the energy harvesting operation performed at R, we assume that R harvests the energy received by antenna 1 during Phase II. This consist of energy carried by signal $x_{s,2}$, transmitted by S, as well as energy transmitted by antenna 2 of R. More specifically, the signal received by antenna 1 of R is expressed as

$$y_{r,2} = \sqrt{P_{s,2}}h_{sr}x_{s,2} + \sqrt{P_r}h_{21}\frac{y_{r,1}}{\sqrt{A}} + n_{r,2}, \tag{5}$$

where h_{21} is the channel formed between antenna 2 and antenna 1 of R, that is hereby assumed to be deterministic. The energy that the relay can harvest is then expressed as [4]

$$E_r = \frac{T}{2}\eta|h_{sr}|^2\left(\sqrt{P_{s,2}} + \frac{\sqrt{P_r P_{s,1}}}{\sqrt{A}}|h_{21}|\right)^2 \tag{6}$$

where η is the efficiency of energy harvesting. The result in (6) can be achieved if $x_{s,2} = x_{s,1}\exp\left(-j\phi_{21}\right)$, where, as in [4], ϕ_{21} is selected such that $\exp\left(-j\phi_{21}\right) = \frac{h_{21}}{|h_{21}|}$. We therefore adopt this choice for $x_{s,2}$. Moreover, since R is powered only by the energy that it harvests, one needs to select P_r such that the energy spent for transmission is at most equal to the harvested energy. This constraint leads to selecting P_r such that $P_r \leq \frac{E_r}{T/2}$. With the specific choice of $x_{s,2}$, aiming at maximizing the achievable rate, we propose selecting P_r such as to exploit all harvested energy, i.e. such that, $P_r\left(T/2\right) = E_r$, which leads to the solution

$$P_r = \frac{\eta|h_{sr}|^2 P_{s,2}}{\left(1 - \rho\sqrt{\beta}\right)^2} \tag{7}$$

where $\rho = \sqrt{\eta}h_{21}$, and $\beta = \frac{A - \sigma_d^2}{A}$. The signal reaching D is then written as

$$
\begin{aligned}
y_{d,2} &= \left(\sqrt{\frac{P_{s,1}P_r}{A}}h_{sr}h_{rd} + \sqrt{P_{s,2}}h_{sd}\exp\left(-j\phi_{21}\right)\right)x_{s,1} \\
&\quad + \sqrt{\frac{P_r}{A}}h_{rd}n_{r,1} + n_{d,2},
\end{aligned}
\tag{8}
$$

or equivalently, substituting (7) in (8), as

$$y_{d,2} = \sqrt{P_{s,2}} \left(\frac{\sqrt{\eta\beta}h_{sr}h_{rd}}{(1-\rho\sqrt{\beta})} + h_{sd}\exp\left(-j\phi_{21}\right) \right) x_{s,1}$$
$$+ \sqrt{P_r/A}h_{rd}n_{r,1} + n_{d,2}. \tag{9}$$

Expression (9) implies that the choice $x_{s,2} = x_{s,1}\exp\left(-j\phi_{21}\right)$, not only allows for maximizing the energy that R can harvest, but also for exploiting the S-D link, during the second phase of the time slot. Moreover, based on (9), the SNR of signal $y_{d,2}$ is expressed as

$$\gamma_{d,2} = \frac{AP_{s,2}\left|\sqrt{\eta\beta}h_{sr}h_{rd} + h_{sd}\exp\left(-j\phi_{21}\right)\left(1-\rho\sqrt{\beta}\right)\right|^2}{\sigma_d^2 A\left(1-\rho\sqrt{\beta}\right)^2 + \eta\sigma_r^2 g_{rd}g_{sr}P_{s,2}}, \tag{10}$$

where $g_{sr} = |h_{sr}|^2$, $g_{rd} = |h_{rd}|^2$.

Applying Maximum Ratio Combining (MRC) in order to combine the signals received by D in the two phases, it is easy to show that the SNR on the resulting channel is expressed as

$$\gamma = \gamma_{sd} + \gamma_{d,2}. \tag{11}$$

As a result, one can write the communication rate of the resulting communications channel as

$$R = \frac{1}{2}\log_2\left(1+\gamma\right). \tag{12}$$

In the following section, we derive an algorithm for solving the problem of optimal power allocation, i.e. determining the optimal value of $P_{s,1}$ and $P_{s,2}$, subject to a total transmit power constraint at S, such as to maximize the achievable communications rate of the cooperative system.

III. Optimal dynamic power allocation

We assume that S has full knowledge of CSI h_{sd}, h_{sr} and h_{rd}. In such a case, we assume that S selects power levels $P_{s,1}$ and $P_{s,2}$ such as to maximize the instantaneous achievable rate, i.e. by solving the following optimization problem:

$$\begin{aligned} \underset{P_{s,1},P_{s,2}}{\text{maximize}} : \quad & \log_2\left(1+\gamma\right) \\ \text{subject to}: \quad & P_{s,1}, P_{s,2} \geq 0, P_{s,1} + P_{s,2} = P, \end{aligned} \tag{13}$$

where P stands for a total transmit power constraint. Equivalently power levels $P_{s,1}$ and $P_{s,2}$ can be found by solving optimization problem

$$\begin{aligned} \underset{P_{s,1},P_{s,2}}{\text{maximize}} : \quad & \gamma \\ \text{subject to}: \quad & P_{s,1}, P_{s,2} \geq 0, P_{s,1} + P_{s,2} = P. \end{aligned} \tag{14}$$

In order to solve optimization problem (14), let us start by introducing the variables

$$\delta_1 = h_{sd}\exp\left(-j\phi_{21}\right), \text{ and } \delta_2 = \sqrt{\eta}h_{sr}h_{rd} - \rho\delta_1. \tag{15}$$

We can then express the SNR γ as

$$\gamma = \frac{g_{sd}P_{s,1}}{\sigma_d^2} + \frac{AP_{s,2}\left(|\delta_1|^2 + 2\Re\{\delta_1\delta_2^\star\}\sqrt{\beta} + |\delta_2|^2\beta\right)}{\sigma_d^2 A\left(1-\rho\sqrt{\beta}\right)^2 + \eta\sigma_r^2 g_{rd}g_{sr}P_{s,2}}. \tag{16}$$

Solving optimization problem (14) in closed form, based on expression (16), proves to be cumbersome. Therefore in what follows we present a procedure for solving the optimal power allocation problem based on using a tight approximation for the SNR γ.

A. Approximating the SNR of the system

Our method for approximating the SNR of the system is based on substituting $\sqrt{\beta}$ in (16), by a properly selected polynomial expansion. In more detail, by noticing that β is upper bounded by

$$\beta_{max} = \frac{g_{sr}P}{g_{sr}P + \sigma_r^2}, \tag{17}$$

we propose approximating $\sqrt{\beta}$, as

$$\sqrt{\beta} \approx \sum_{n=0}^{N} c_n\beta^n, \beta \leq \beta_{max}. \tag{18}$$

where coefficients $c_n, n = 0, \dots, N$, are selected such as to minimize the mean squared error, defined as

$$\mathcal{E} = \int_0^{\beta_{max}} \left(\sqrt{y} - \sum_{n=0}^{N} c_n y^n\right)^2 dy. \tag{19}$$

It is then easy to show that coefficients $c_n, n = 0, \dots, N$, are the elements of the vector $\mathbf{c} = [c_0, c_1, \dots, c_N]^T$, which comes from solving the system

$$\mathbf{Ac} = \mathbf{z} \tag{20}$$

where

$$\mathbf{A}[k,l] = \int_0^{\beta_{max}} y^{k+l-2}dy = \frac{\beta_{max}^{k+l-1}}{k+l-1}, \tag{21}$$

and

$$\mathbf{z}[k] = \int_0^{\beta_{max}} y^{k-1/2}dy = \frac{\beta_{max}^{k+1/2}}{k+1/2}. \tag{22}$$

Using the above approximation for $N = 2$, and the definition of β, we can then obtain the approximations given in (23). As a result, substituting (23) in (16) and setting $P_{s,2} = P - P_{s,1}$, we obtain the following approximation

$$\tilde{\gamma} = f\left(P_{s,1}\right) = \gamma_{sd} + \left(P - P_{s,1}\right)\frac{e_2 P_{s,1}^2 + e_1 P_{s,1} + e_0}{f_2 P_{s,1}^2 + f_1 P_{s,1} + f_0}, \tag{24}$$

where

$$\begin{aligned} e_2 &= \left(|\delta_1|^2 + 2\Re\{\delta_1\delta_2^\star\}D_2 + |\delta_2|^2\right)g_{sr}^2 \\ e_1 &= \left(2|\delta_1|^2 + 2\Re\{\delta_1\delta_2^\star\}D_1 + |\delta_2|^2\right)\sigma_r^2 g_{sr} \\ e_0 &= \left(|\delta_1|^2 + 2\Re\{\delta_1\delta_2^\star\}D_0\right)\sigma_r^4 \end{aligned} \tag{25}$$

and

$$\begin{aligned} f_2 &= \left(\sigma_d^2\left(1+\rho^2 - 2\rho D_0\right) - \eta\sigma_r^2 g_{rd}\right)g_{sr}^2, \\ f_1 &= \left(\sigma_d^2\left(2+\rho^2 - 2\rho D_1\right) + \eta g_{rd}\left(g_{sr}P - \sigma_r^2\right)\right)\sigma_r^2 g_{sr}, \\ \text{and } f_0 &= \sigma_d^2(1-2\rho D_0)\sigma_r^4 + \eta\sigma_r^4 g_{rd}g_{sr}P, \end{aligned} \tag{26}$$

$$\left(1 - \rho\sqrt{\beta}\right)^2 = 1 + \rho^2\beta - 2\rho\sqrt{\beta} \approx \frac{\left(1 + \rho^2 - 2\rho D_2\right) g_{sr}^2 P_{s,1}^2 + \left(2 + \rho^2 - 2\rho D_1\right) \sigma_r^2 g_{sr} P_{s,1} + \left(1 - 2\rho D_0\right) \sigma_r^4}{\left(g_{sr} P_{s,1} + \sigma_r^2\right)^2},$$

$$\text{and} \quad \sqrt{\beta} \approx \frac{D_2 g_{sr}^2 P_{s,1}^2 + D_1 \sigma_r^2 g_{sr} P_{s1} + D_0 \sigma_r^4}{\left(g_{sr} P_{s,1} + \sigma_r^2\right)^2}, \quad \text{with} \quad D_0 = c_0, \quad D_1 = c_1 + 2c_0, \quad D_2 = c_0 + c_1 + c_2 \tag{23}$$

Therefore, taking into account the fact that multiplication of polynomials $P - P_{s,1}$ and $e_2 P_{s,1}^2 + e_1 P_{s,1} + e_0$ corresponds to convolution of their coefficients, we can rewrite (24), as

$$\tilde{\gamma} = f\left(P_{s,1}\right) = \frac{g_3 P_{s,1}^3 + g_2 P_{s,1}^2 + g_1 P_{s,1} + g_0}{f_2 P_{s,1}^2 + f_1 P_{s,1} + f_0}, \tag{27}$$

where coefficients $g_n, n = 0, \ldots, 3$, are found to be the elements of the vector $\mathbf{g} = [g_3, g_2, g_1, g_0]^T$, defined as

$$\mathbf{g} = [-1, P]^T * \mathbf{e} + \frac{g_{sd}}{\sigma_d^2} [f_2, f_1, f_0, 0]^T, \tag{28}$$

with $\mathbf{e} = [e_2, e_1, e_0]$. In the following section, we exploit approximation $\tilde{\gamma}$ such as to solve the optimal power allocation problem.

B. Solving the optimal power allocation problem

Capitalizing on the rational approximation (27), we propose using the power allocation that is derived by solving the following optimization problem:

$$\underset{P_{s,1}, P_{s,2}}{\text{maximize}} : \quad \tilde{\gamma} \tag{29}$$
$$\text{subject to: } 0 \leq P_{s,1}, \leq P.$$

This can be solved by finding the roots of equation

$$\frac{df\left(P_{s,1}\right)}{dP_{s,1}} = 0. \tag{30}$$

or equivalently of equation:

$$g_3 f_2 P_{s,1}^4 + 2g_3 f_1 P_{s,1}^3 + \left(3g_3 f_0 + g_2 f_1 - g_1 f_2\right) P_{s,1}^2 + 2g_2 f_0 P_{s,1} + g_1 f_0 = 0. \tag{31}$$

Let $\mathcal{P} = \{P_1, P_2, P_3, P_4\}$ be the roots of the polynomial (31). Moreover, let the set $\mathcal{P}'$ be defined as

$$\mathcal{P}' = \{0, P, P_i | P_i \in \mathcal{P}, 0 \leq P_i \leq P\}. \tag{32}$$

The solution to optimization problem (29) is then found by selecting out of the elements of set $\mathcal{P}'$, the one that maximizes the cost function of (29). Note that roots of equation (31) can be easily found using standard methods for solving quartic equations [11, eq 3.8.3, pp. 17].

IV. NUMERICAL RESULTS

In this section, we present performance analysis results obtained applying our derived power allocation. Before that, in order to validate the accuracy of the proposed approximation, in Fig. 1 we present the relative error for approximation $\tilde{\gamma}$ for different values of $P_{s,1}$, for a selected channel realization. In more detail, in Fig. 1, we have set the noise variance at R and D to be equal to $\sigma_r^2 = \sigma_d^2 = 1$. Moreover, we have
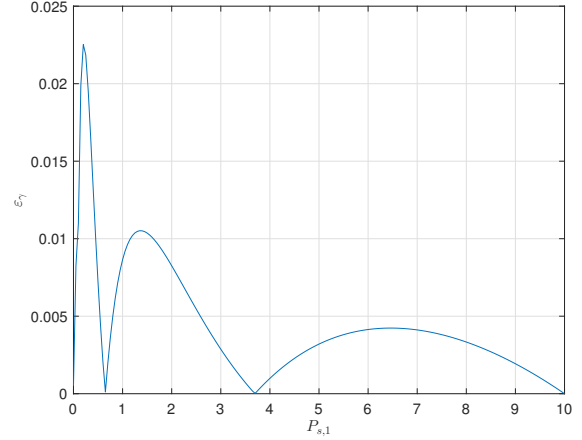

Fig. 1. Relative error for approximation $\tilde{\gamma}$

set the energy harvesting efficiency to be equal to $\eta = 0.8$, and, similarly to [4], we have set the energy recycling channel amplitude h_{21}, such that $10 \log_{10} |h_{21}|^2 = -15[dB]$, and $\phi_{21} = 0$. The total power constraint P was set equal to $P = 10\sigma_d^2$. Moreover, we have set the S-D, S-R and R-D channel gains to be equal to

$$h_{sr} = h_{rd} = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}, \quad \text{and} \quad h_{sd} = \frac{1}{10} + j\frac{1}{10}. \tag{33}$$

The particular choice of values corresponds to a case that the S-R and R-D links are of unit amplitude, and the gain on the S-D link is substantially weaker than the gain on the S-R and R-D links. The results shown in Fig. 1 validate that approximation $\tilde{\gamma}$ closely matches the exact value of γ and the relative approximation error is less than 10^{-2} for the majority of values for $P_{s,1}$, while its maximum value is less than 0.025. As a result, we can conclude that solving optimization problem (29) can lead us to a power allocation close to the optimal power allocation. Moreover, problem (29) can be solved using standard algebraic techniques, as opposed to problem (14) that cannot be solved analytically and thus requires using iterative optimization techniques, which may lead to increased complexity.

In order to illustrate the benefits of the derived policy, we now plot its achievable performance in terms of ergodic capacity, and we compare it with the ergodic capacity for the case that $P_{s,1} = P_{s,2} = P/2$. In more detail, we consider the system shown in Fig. 2, where d_{sr} is the distance between the source and the relay, d_{rd} is the distance between the relay and the destination, and d_{sd} the distance between S and D.
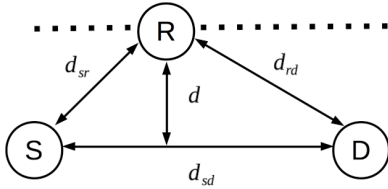
Fig. 2. The considered system model

Finally, d is the distance between R and the S-D path, where all distances are expressed in meters.

Assuming that the S-R and R-D channels are subject to pathloss and Rayleigh fading, and adopting a power-law model for pathloss, we can write the channel gains g_{sr}, and g_{rd} as

$$g_{sr} = \frac{\alpha \left| \tilde{h}_{sr} \right|^2}{d_{sr}^m}, \text{and} \ \ g_{rd} = \frac{\alpha \left| \tilde{h}_{rd} \right|^2}{d_{rd}^m}, \qquad (34)$$

where α is an attenuation factor, m is the pathloss exponent, $\tilde{h}_{sr} \sim \mathcal{CN}(0,1)$ and $\tilde{h}_{rd} \sim \mathcal{CN}(0,1)$. Moreover, we also set h_{sd} to be a Rayleigh fading channel, i.e., it holds that $h_{sd} \sim \mathcal{CN}(0,\epsilon)$, where we select ϵ small enough, e.g., $\epsilon = 0.01 \min \left\{ \frac{\alpha}{d_{sr}^m}, \frac{\alpha}{d_{rd}^m} \right\}$, such as to emulate a scenario where the S-D link is weak, thus requiring the assistance of the S-R-D link. For this specific system model, in Fig. 3 we present the achievable ergodic rate as a function of d_{sr} along with the achievable ergodic rate, in case of a fixed power allocation, i.e., in case that $P_{s,1} = P_{s,2} = P/2$. For these simulations, the distance between S and D was set to be equal to $d_{sd} = 50m$. Moreover, the distance d between the relay R and the S-D line of sight path, was set to be equal to $d = d_{sd}/4$. Parameters $\sigma_d^2, \sigma_r^2, \eta$, and h_{21} were set to the values presented earlier. Moreover, for simplicity we had set $\alpha = 1$ and $m = 2$. Concerning the value of P we had set it such that

$$\frac{P}{2d_{sr}^2} = 10^{1.5}, \qquad (35)$$

i.e., such that the average SNR on the S-R link is equal to 15dB in both phases of transmission, in case that $P_{s,1} = P_{s,2} = P/2$. Concerning the position of the relay R, we have considered several different placements for R, on the line segment shown with the dotted line in Fig. 2 that is parallel to the S-D path. That is, considering S to be the origin of our coordinate system, and denoting the position of D as $(x_d, y_d) = (d_{sd}, 0)$, we have considered different placements of R, where the coordinates (x_r, y_r) are constrained by the following rules:

$$0 \leq x_r \leq d_{sd}, \text{ and } \ y_r = d. \qquad (36)$$

From Fig. 3 we observe that the proposed power allocation results in substantial performance benefits, compared to the case of a fixed power allocation, especially as the distance d_{sr} increases, or equivalently, as R is placed closer to D.
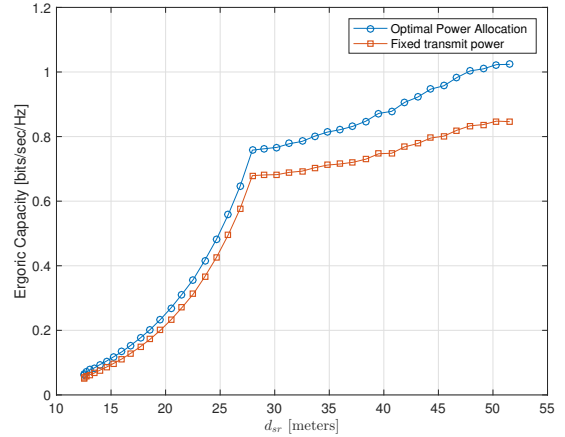

Fig. 3. Performance comparison of the derived allocation scheme with the performance of fixed power allocation

V. CONCLUSION

The problem of the optimal design of power allocation for a full duplex relaying system that exploits energy harvesting and energy recycling was considered. For this system, we have presented a technique for approximating the SNR of the system and the accuracy of this approximation was validated. Based on this approximation, we have developed a power allocation scheme that allows for determining the power level values for the source of the relaying system for the two phases of the communication scheme, that can be analytically solved by finding the roots of a quartic formula. The performance of this power allocation scheme has been compared with the performance achievable when the source transmits using the same power level in both phases of the transmission and considerable performance improvement was noted.

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