Joint Power and Sensing Optimization for Hybrid Cognitive Radios with limited CSIT

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***Abstract*—The problem of optimal Power Policy (PP) design for hybrid interweave/underlay Cognitive Radio (CR) systems is investigated, in the presence of imperfect Spectrum Sensing (SS). A limited Channel State Information at the Transmitter (CSIT) scenario is considered, that has never been studied before, where the Secondary Transmitter (STx) uses combined instantaneous and statistical CSIT. Given such CSIT, the optimal PP with the aim of maximizing the average transmission rate of the CR system, subject to a constraint on the average interference caused by STx transmission to the primary system, is found. The derived PP is then exploited for the development of an iterative framework for the problem of jointly optimizing the sensing time and the applied PP. By means of simulations, it is shown that the proposed joint SS and PP optimization framework offers a clear gain in terms of the achievable rate of the CR system, with respect to conventional underlay and interweave CR systems, especially for intermediate values of the average interference constraint and also for different primary user activity regimes.**

I. Introduction

Cognitive Radio (CR) has emerged as an efficient way to utilize the bandwidth of wireless communications systems. The key idea that the CR concept is based upon, is allowing non-licensed users, i.e., Secondary Transmitter (STx)-Secondary Receiver (SRx) pairs, to exploit spectral resources principally licensed to a Primary Transmitter (PTx)-Primary Receiver (PRx) pair. To this end, three different approaches (overlay/underlay/interweave), described in [1], have been investigated such as to cope with the issues rising from the allocation of the same spectral resources to PTxs and STxs. Among these pre-described flavors of CR systems, underlay and interweave CR approaches have been considered as particularly attractive solutions. Starting from the case of underlay CR, among the several research topics that have been examined, considerable efforts have been concentrated on the problem of optimal Power Policy (PP) design [2]–[7], such as to maximize the average communication rate of the secondary system, subject to a constraint on the average interference caused by STx to PRx. A common characteristic of the previous policies is the fact that they are based on the assumption of constant presence of a PTx at the system. Nonetheless, such an assumption is unrealistic, since in practical systems, PTxs are characterized by activity profiles, and so-called "spectrum holes", i.e., time intervals where licensed spectrum is not used by PTx, are expected to appear [8].

On the other hand, in the case of interweave CR, the main idea is solely the exploitation of these spectrum holes. For the detection of the latter, the use of Spectrum Sensing (SS) techniques is required. Due to its simplicity and ease of implementation, one of the most common SS techniques is the well-known Energy Detection (ED) [9]. In the case of interweave CR, several works [10]–[12] study the interesting tradeoff that appears between allocating higher sensing time such as to increase sensing reliability hence avoid missed detection and false alarm events, and keeping sensing time small such as to minimize the STx-SRx communication rate loss due to the sensing procedure. This problem is jointly treated with the problem of optimal PP design. In [13] a different interweave based CR approach is presented where SS and data transmission are performed simultaneously by the CR system. For such a system model, the optimal transmit PP for the CR system is derived, taking into account an average transmit power constraint for STx as well as a constraint on the average interference caused by the CR system to the primary system.

In an effort to maximize the spectral efficiency of CR spectrum access, in several works, [14]–[16], the problem of PP design for Single-Input-Single-Output (SISO) systems based on the combined application of the interweave and underlay techniques has been considered. However, in these works, instantaneous CSIT for the interference channel links between the primary and secondary system is assumed to be available at STx. Such a strong assumption is difficult to be met in practice, since it requires coordination between the primary and secondary network which may not be available [7]. Moreover, among the aforementioned papers, the problem of SS parameter optimization is treated only in [14].

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Motivated by the above, in this work, we focus on the Single-Input-Multiple-Output (SIMO) system model that has not been studied in the past, and we consider the problem of joint PP and sensing time optimization in hybrid interweave/underlay CR networks by taking into account more realistic assumptions concerning CSIT availability at STx. Unlike [14]–[16], we assume that STx has instantaneous knowledge concerning only the STx-SRx link as well as statistical CSIT for the STx-PRx and PTx-SRx links. Given these assumptions, the contribution of the paper is summarized as follows:

- Initially, the problem of PP optimization for hybrid CR systems based on the combination of interweave/underlay CR is considered. The aim of such a PP is to maximize the rate of secondary communication, taking into account the imperfect SS, subject to constraints regarding the Quality-of-Service (QoS) for primary communication. The optimum PP for solving this problem is derived and a low complexity algorithm for its implementation is proposed.
- Following that, capitalizing on the derived PP, an iterative framework is introduced such as to provide suboptimum solutions to the joint SS and PP optimization problem.
- Based on Monte Carlo simulations, it is shown that the proposed joint SS and PP optimization framework offers clear gains, in terms of the achievable rate of the CR system, with respect to conventional underlay/interweave CR systems operating with the same CSIT assumptions. The gains are particularly notable for intermediate values of the average interference constraint.

Notation: Throughout the paper the following notations are adopted. We use $E_1(\cdot)$ to denote the exponential integral [17, eq. 5.1.1]. The Gauss Q function is denoted as $\mathcal{Q}(x) = \frac{1}{2}\text{erfc}\left(\frac{x}{\sqrt{2}}\right)$ with $\text{erfc}(\cdot)$ defined in [17, eq. 7.1.2]. Operator $[x]^+$ stands for $\max\{x, 0\}$. Bold lower case letters stand for vectors and bold upper case letters are used to denote matrices. Euclidean norm is denoted by $\|\cdot\|$. The identity matrix of dimension $M \times M$ is denoted as $\mathbf{I}_M$. Moreover, notation $\mathbf{x} \sim (\mathbf{0}_M, \mathbf{R})$ is used to indicate that the $M \times 1$ vector $\mathbf{x}$ follows a Circularly Symmetric Complex Gaussian (CSCG) distribution with zero mean and covariance matrix $\mathbf{R}$. We use the notation $f_{x_1,\ldots,x_n}(\cdot,\ldots,\cdot)$ to define the joint Probability Density Function (PDF) of Random Variables (RVs) $x_1,\ldots,x_n$, and $E_{x_1,\ldots,x_n}\{\phi(x_1,\ldots,x_n)\}$ to denote the expected value of function $\phi(x_1,\ldots,x_n)$ where expectation is taken with respect to the distribution of RVs $x_1,\ldots,x_n$.

II. System model

A SIMO CR system with M receive antennas is considered, that coexists with a SISO primary system[1]. Similar to [18], [19], we assume that SS is applied at STx in order to detect the presence of primary transmission. Thus, the Medium Access Control (MAC) frame of the system, assumed to be of duration of T time slots, is divided in a sensing subframe, of duration of $\tau \leq T$ time slots, during which STx applies SS, and a data transmission subframe, during which STx transmits data to SRx. For the description of the system operation during these two subframes, the following events need to be defined: 1) Event $\mathcal{H}_0$ that corresponds to the case that the communication channel is not used by the PTx, 2) Event $\hat{\mathcal{H}}_0$ that occurs when no primary user is detected by the STx during the SS period, as well as their complementary events, denoted as $\mathcal{H}_1$ and $\hat{\mathcal{H}}_1$, respectively.

[1]Extension to SIMO primary systems is straightforward.

A. Sensing subframe

The application of ED based SS at STx is considered. In more detail, we assume that STx senses the wireless channel by sampling the received signal at instances that are multiples of $1/f_s$, where f_s stands for the sampling frequency of SS. As a result, ED is based on $N = \tau f_s$ samples, assuming that τ is an integer multiple of $1/f_s$. We can write the signal sensed by STx at any of these instances as

$$r_s[n] = \begin{cases} \eta[n], & \text{if } \mathcal{H}_0 \\ h_t\sqrt{P_p}s_p[n] + \eta[n], & \text{if } \mathcal{H}_1, \end{cases} \tag{1}$$

$n \leq N$, where $\eta[n] \sim \mathcal{CN}(0, N_{0,t})$ stands for the complex Additive White Gaussian Noise (AWGN) at STx during ED, $s_p[n] \sim \mathcal{CN}(0,1)$ is the signal transmitted by PTx, P_p is the power transmitted by PTx that is assumed to be fixed and known to STx, and h_t is the complex valued PTx-STx fading channel, assumed to be constant during a MAC frame. In our analysis, we assume that the PTx-STx channel is a Rayleigh fading channel, i.e., it holds that $h_t \sim \mathcal{CN}\left(0, \sigma_t^2\right)$.

Under these assumptions, when an ED SS scheme is applied, characterized by a decision threshold ε, one can write the probability of false alarm as [18], [20]

$$\mathcal{P}_f(N, \varepsilon) = \Pr\left(\hat{\mathcal{H}}_1 | \mathcal{H}_0\right) = \mathcal{Q}\left(\sqrt{N}\left(\frac{\varepsilon}{N_{0,t}} - 1\right)\right). \tag{2}$$

While $\mathcal{P}_f$ depends exclusively on noise statistics, in the absence of instantaneous CSIT for h_t at STx, the probability of detection, $\mathcal{P}_d$, is expressed by taking into account the statistics of the Signal to Noise Ratio (SNR) on the PTx-STx link. In more detail, by defining the PTx-STx SNR as $\gamma_t = \frac{|h_t|^2 P_p}{N_{0,t}}$, we can express $\mathcal{P}_d$ in the form

$$\mathcal{P}_d(N, \varepsilon) = \Pr\left(\hat{\mathcal{H}}_1 | \mathcal{H}_1\right) = \int_0^\infty \mathcal{P}_{d|\gamma_t}(N, \varepsilon, \gamma_t) f_{\gamma_t}(\gamma_t)\, d\gamma_t, \tag{3}$$

where $\mathcal{P}_{d|\gamma_t}(\gamma_t)$ is the detection probability given γ_t, expressed as [18], [20]

$$\mathcal{P}_{d|\gamma_t}(N, \varepsilon, \gamma_t) = \mathcal{Q}\left(\sqrt{N}\left(\frac{\varepsilon}{N_{0,t}(1+\gamma_t)} - 1\right)\right). \tag{4}$$

Thus, exploiting the fact that due to the Rayleigh fading assumption for channel h_t, it holds that

$$f_{\gamma_t}(\gamma_t) = \frac{1}{\bar{\gamma}_t}\exp\left(-\frac{\gamma_t}{\bar{\gamma}_t}\right), \tag{5}$$

where $\bar{\gamma}_t = \frac{\sigma_t^2 P_p}{N_{0,t}}$, one can easily and accurately approximate the average (over fading) detection probability $\mathcal{P}_d$ by employing the Gauss-Laguerre quadrature rule [17, eq. 25.4.45]. Having presented the SS subframe, in what follows we describe system operation during the data transmission phase of the MAC frame.

B. Data transmission subframe

In our analysis, we consider a hybrid CR system that combines the interweave and underlay approaches. That is, we consider an approach where the transmit power of STx varies as a function of the decision of SS as well as of the available CSIT. In more detail, we consider that an underlay approach is applied, i.e., $P_s = P_{s,1}$ in case that SS decides upon the existence of a primary user, i.e., when event $\hat{\mathcal{H}}_1$ occurs, and that $P_s = P_{s,0}$, in case that SS decides upon the absence of a primary user, i.e., when event $\hat{\mathcal{H}}_0$ occurs, and an interweave communication approach is adopted. As a result, given that event $\hat{\mathcal{H}}_k, k \in \{0,1\}$ occurs, we can write the signal received by SRx during any time slot of the data transmission phase of the MAC frame as

$$y_{s,k} = \begin{cases} \mathbf{v}^H \mathbf{h}_{ss} \sqrt{P_{s,k}} s_s + \mathbf{v}^H \mathbf{n}, & \text{if } \mathcal{H}_0, \hat{\mathcal{H}}_k \\ \mathbf{v}^H \mathbf{h}_{ss} \sqrt{P_{s,k}} s_s + \mathbf{v}^H \mathbf{h}_{ps} \sqrt{P_p} s_p + \mathbf{v}^H \mathbf{n}, & \text{if } \mathcal{H}_1, \hat{\mathcal{H}}_k, \end{cases} \tag{6}$$

$k \in \{0,1\}$, where $\mathbf{h}_{ss}$ denotes the STx-SRx SIMO channel, assumed to be known to SRx, $\mathbf{h}_{ps}$ denotes the PTx-SRx channel, that is known only statistically to SRx, and $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, N_{0,r}\mathbf{I}_M)$ denotes the AWGN at the output of the M receive antennas. In addition, $s_p \sim \mathcal{CN}(0,1)$ and $s_s \sim \mathcal{CN}(0,1)$ denote the signals transmitted by PTx and STx respectively, while $\mathbf{v}$ denotes the unit norm combiner applied at SRx. In what follows, we assume that the applied combiner $\mathbf{v}$ is the Maximum Ratio Combiner (MRC), that has the form $\mathbf{v} = \frac{\mathbf{h}_{ss}}{\|\mathbf{h}_{ss}\|}$. The reasoning behind this choice lies on the fact that MRC is a popular and easy to implement combiner. By adopting MRC, the achievable rate of the investigated CR system for the two applied approaches is expressed as follows.

1) Interweave approach: This approach is followed in case that event $\hat{\mathcal{H}}_0$ is encountered. In this case, the achievable instantaneous rate of the STx-SRx communication link, measured in $bits/sec/Hz$ is expressed as

$$C_0 = \alpha_0 \log_2\left(1 + \frac{gP_{s,0}}{N_{0,r}}\right) + \beta_0 \log_2\left(1 + \frac{gP_{s,0}}{N_{0,r} + w}\right), \tag{7}$$

where

$$\alpha_0 = \frac{(T-\tau)\mathcal{P}_0(1-\mathcal{P}_f)}{T}, \text{and } \beta_0 = \frac{(T-\tau)\mathcal{P}_1(1-\mathcal{P}_d)}{T}, \tag{8}$$

with $\mathcal{P}_0 = \Pr(\mathcal{H}_0)$ and $\mathcal{P}_1 = \Pr(\mathcal{H}_1) = 1 - \mathcal{P}_0$. Moreover, quantities g and w are defined as $g = \|\mathbf{h}_{ss}\|^2$ and $w = \frac{|\mathbf{h}_{ss}^H \mathbf{h}_{ps}|^2 P_p}{\|\mathbf{h}_{ss}\|^2}$.

2) Underlay approach: This approach is followed in case that event $\hat{\mathcal{H}}_1$ is encountered. In such a case, the achievable instantaneous rate of the STx-SRx communication link is expressed as

$$C_1 = \alpha_1 \log_2\left(1 + \frac{gP_{s,1}}{N_{0,r}}\right) + \beta_1 \log_2\left(1 + \frac{gP_{s,1}}{N_{0,r} + w}\right), \tag{9}$$

where α_1 and β_1 are defined as

$$\alpha_1 = \frac{(T-\tau)\mathcal{P}_0\mathcal{P}_f}{T}, \text{and } \beta_1 = \frac{(T-\tau)\mathcal{P}_1\mathcal{P}_d}{T}. \tag{10}$$

Moreover, regardless of the applied approach, we adopt an independent identically distributed (i.i.d.) Rayleigh fading channel model for channels $\mathbf{h}_{ss}$ and $\mathbf{h}_{ps}$, i.e., we assume that $\mathbf{h}_{ss} \sim \mathcal{CN}(\mathbf{0}, \sigma_{ss}^2 \mathbf{I}_M)$, and $\mathbf{h}_{ps} \sim \mathcal{CN}(\mathbf{0}, \sigma_{ps}^2 \mathbf{I}_M)$. In addition, we assume that STx has knowledge of quantities $\mathcal{P}_0, \mathcal{P}_d, \mathcal{P}_f, N_{0,r}$, and σ_{ps}^2, instantaneous feedback regarding g as well as knowledge of its statistics, i.e., its PDF. Given such knowledge, we are initially interested in the design of the optimal power policies $P_{s,0}(g)$ and $P_{s,1}(g)$ such as to maximize the average rate of the secondary communication system, subject to a primary QoS constraint, expressed in terms of the average interference caused by STx transmission to PRx reception. This optimization problem is formally exposed in the following section.

III. Problem Formulation

Based on (7) and (9), one can write the average achievable rate of secondary communication for any of the applied approaches as

$$\begin{aligned} \mathcal{C}_k &= E_{g,w}\left\{C_k\left(g, P_{s,k}(g), w\right)\right\} \\ &= \int_0^\infty \int_0^\infty C_k\left(g, P_{s,k}(g), w\right) f_{g,w}(g,w)\, dw dg, \end{aligned} \tag{11}$$

$k \in \{0,1\}$, where we assume that the applied PP $P_{s,k}(g), k \in \{0,1\}$ is based on instantaneous knowledge of g and only statistical knowledge of w, i.e., $\mathbf{h}_{ps}$ is considered to be known only statistically. Thus, one can write the optimization problem that we wish to solve as

$$\begin{aligned} &\underset{P_{s,k}(g), k\in\{0,1\}}{\text{maximize}} \ : \ \mathcal{C} = \mathcal{C}_0 + \mathcal{C}_1 \\ &\text{subject to:} \quad (1-\mathcal{P}_d) E_g\{P_{s,0}(g)\} + \mathcal{P}_d E_g\{P_{s,1}(g)\} \le \bar{P} \\ &\text{and} \quad 0 \le P_{s,0}(g), P_{s,1}(g) \le P_{peak}, \end{aligned} \tag{12}$$

where, following an approach similar to the one presented in [2]–[4], [7], constraint $\bar{P}$ is selected such that an average interference constraint $\mathcal{I}$ caused by STx transmission to PRx reception, is achieved, i.e., $E_{h_{sp}}\left\{|h_{sp}|^2 \bar{P}\right\} \le \mathcal{I}$ where h_{sp} denotes the STx-PRx fading channel. Clearly, determination of $\bar{P}$ requires knowledge of $E\left\{ |h_{sp}|^2 \right\}$. The use of such average interference constraints has been proposed in several cases in the CR technical literature, e.g., [2]- [7].

In the absence of instantaneous feedback for w, in order to solve problem (12), one needs to start by calculating the expectation of the rate with respect to w, i.e., by deriving an expression for the inner integral in (11). This can be done by noticing that under the assumption that $\mathbf{h}_{ps} \sim \mathcal{CN}(\mathbf{0}, \sigma_{ps}^2 \mathbf{I}_M)$, RVs g and w are independent and w follows an exponential distribution with mean value $\bar{w} = \sigma_{ps}^2 P_p$ [21]. Thus, by employing [7, eq. 3] one can rewrite $\mathcal{C}_k$ as

$$\begin{aligned} \mathcal{C}_k = &\frac{\alpha_k + \beta_k}{\ln 2} \int_0^\infty \ln\left(1 + \frac{gP_{s,k}(g)}{N_{0,r}}\right) f_g(g) dg \\ &+ \frac{\beta_k}{\ln 2} \int_0^\infty U\left(gP_{s,k}(g)\right) f_g(g)\, dg - \frac{\beta_k}{\ln 2} U(0), \end{aligned} \tag{13}$$

$k \in \{0,1\}$, where [7]

$$U(x) = \exp\left(\frac{N_{0,r} + x}{\bar{w}}\right) E_1\left(\frac{N_{0,r} + x}{\bar{w}}\right), x \ge 0. \tag{14}$$

As a result, it is easy to show that the optimization problem (12) is equivalent to the following optimization problem

$$\begin{aligned}
&\underset{P_{s,k}(g),k\in\{0,1\}}{\text{minimize}} : \\
&-\sum_{k=0}^{1} E_g\left\{(\alpha_k+\beta_k)\ln\left(1+\frac{gP_{s,k}(g)}{N_{0,r}}\right)+\beta_k U\left(gP_{s,k}\left(g\right)\right)\right\} \\
&\text{subject to:} \quad \sum_{k=0}^{1}\pi_k E_g\left\{P_{s,k}\left(g\right)\right\}\leq \bar{P}, \\
&\text{and} \quad 0\leq P_{s,0}\left(g\right), P_{s,1}\left(g\right)\leq P_{peak},
\end{aligned} \tag{15}$$

where $\pi_0 = 1-\mathcal{P}_d$ and $\pi_1 = \mathcal{P}_d$. In the following section, the optimal PP $P_{s,k}(g), k \in \{0,1\}$ for maximizing the average rate of secondary communication is derived by solving optimization problem (15).

IV. Ergodic rate maximization policy

The derivation of the optimal PP for maximizing the ergodic rate of secondary communication, i.e., solving optimization problem (15), can be performed by exploiting its convexity. The convexity of the objective function can be easily proven by noticing that function $-\ln\left(1+\frac{gP_{s,k}}{N_{0,r}}\right)$ is convex with respect to $P_{s,k}$, and using property [17, eq. 5.1.19] in order to prove the convexity of function $-\ln\left(1+\frac{gP_{s,k}}{N_{0,r}}\right) - U\left(gP_{s,k}\right)$ with respect to $P_{s,k}$, by applying the second derivative test. Thus, the objective function in problem (15) is essentially a positively weighted sum of separable convex functions, each one corresponding to a different fading state and a different SS decision, i.e., $\hat{\mathcal{H}}_0$ or $\hat{\mathcal{H}}_1$. Moreover, the constraints of problem (15) are convex. Exploiting convexity of problem (15), one can easily solve it and obtain the optimal PP in the form given in the following theorem.

Theorem 1: The optimal PP for maximizing the ergodic rate of the secondary system is expressed as

$$P^{\star}_{s,k}\left(g\right)=\min\left\{\left[\frac{1}{g}\mathcal{V}\left(\frac{\lambda\pi_k}{g};\alpha_k,\beta_k\right)\right]^{+}, P_{peak}\right\}, \tag{16}$$

where $\mathcal{V}\left(\cdot;\alpha,\beta\right)$ is defined as the inverse function of $\mathcal{U}\left(x;\alpha,\beta\right)$ that is given by

$$\mathcal{U}\left(x;\alpha,\beta\right)=\frac{\alpha}{N_{0,r}+x}+\frac{\beta}{\bar{w}}U\left(x\right). \tag{17}$$

As a result, it holds that $\mathcal{V}\left(\mathcal{U}\left(x;\alpha,\beta\right)\right) = x$. Regarding parameter λ in (16), this is essentially a non-negative Lagrange multiplier and its value $\lambda = \lambda^{\star}$ is selected such that the constraint $\sum_{k=0}^{1}\pi_k E_g\left\{P_{s,k}\left(g\right)\right\} = \bar{P}$ is satisfied[2].

Proof: It is easy to see that for problem (15), Slater's conditions hold and thus duality gap is zero. Moreover, since the objective function of (15) is a positively weighted sum of separable convex functions, we can solve problem (15) by applying a dual decomposition approach that is described in detail in [22]. For the application of such an approach one needs to introduce the partial Lagrangian function

$$\begin{aligned}
L\left(P_s\left(g\right),\lambda\right) = &-\sum_{k=0}^{1}\left(\alpha_k+\beta_k\right)E_g\left\{\ln\left(1+\frac{gP_{s,k}\left(g\right)}{N_{0,r}}\right)\right\} \\
&-\sum_{k=0}^{1}\beta_k E_g\left\{U\left(gP_{s,k}\right)\right\}+\lambda\left(\sum_{k=0}^{1}\pi_k E_g\left\{P_{s,k}\left(g\right)\right\}-\bar{P}\right),
\end{aligned} \tag{18}$$

where λ is the non negative Lagrange multiplier corresponding to the coupling constraint $\sum_{k=0}^{1}\pi_k E_g\left\{P_{s,k}\left(g\right)\right\} \leq \bar{P}$. One can then solve optimization problem (15), by solving the univariate problem that corresponds to each one of the fading states, for each SS decision, i.e., the problem

$$\begin{aligned}
&\underset{P_{s,k(g)},k\in\{0,1\}}{\text{minimize}} : \\
&-\left(\alpha_k+\beta_k\right)\ln\left(1+\frac{gP_{s,k}(g)}{N_{0,r}}\right)-\beta_k U\left(gP_{s,k}\left(g\right)\right) \\
&+\lambda\pi_k P_{s,k}\left(g\right) \\
&\text{subject to:} \quad 0\leq P_{s,k}\left(g\right)\leq P_{peak},
\end{aligned} \tag{19}$$

$k \in \{0,1\}$, for any given value of λ. Problem (19) is convex and can be solved by introducing Lagrange multipliers $\mu_k\left(g\right)$ and $\nu_k\left(g\right)$ for the peak power and positivity constraints respectively, and applying Karush Kuhn Tucker (KKT) conditions [23]. The optimal policy is then given by solving the system of equations

$$\begin{aligned}
&\frac{\alpha_k g}{N_0+gP_{s,k}\left(g\right)}+\frac{\beta_k g}{\bar{w}}U\left(gP_{s,k}\left(g\right)\right)=\lambda\pi_k+\mu_k\left(g\right)-\nu_k\left(g\right), \\
&\mu_k\left(g\right)\left(P_{s,k}\left(g\right)-P_{peak}\right)=0,\ \nu_k\left(g\right)P_{s,k}\left(g\right)=0, \\
&\mu_k\left(g\right),\nu_k\left(g\right)\geq 0
\end{aligned} \tag{20}$$

that leads to the solution given in equation (16). The value of Lagrange multiplier $\lambda = \lambda^{\star}$ is then chosen such that the coupling constraint is satisfied with equality. ■

Clearly, the result of Theorem 1, holds under the assumption that function $\mathcal{U}\left(\cdot;\cdot,\cdot\right)$ is a monotonous decreasing function, such that its inverse can be defined. This result can be easily proven, by employing inequalities [17, eq. 5.1.19] to prove that $\frac{d\mathcal{U}(x;\alpha,\beta)}{dx} < 0$ for $x \geq 0$ and $\alpha, \beta > 0$. In the following section, a technique is introduced for approximating function $\mathcal{V}\left(\cdot;\cdot,\cdot\right)$ by employing the monotonicity of function $\mathcal{U}\left(x;\alpha,\beta\right)$ along with approximations for this function.

V. Approximations for the optimal PP

As it can be seen from Theorem 1, one can find the optimal PP that should be applied by STx, given knowledge of the fading channel state g, by solving, with respect to x, the equation

$$\mathcal{U}\left(x;\alpha_k,\beta_k\right)=\frac{\lambda\pi_k}{g}, \tag{21}$$

or equivalently, by introducing the auxiliary variable

$$z=\frac{N_{0,r}+x}{\bar{w}}, \tag{22}$$

[2]Clearly, such a constraint can be satisfied provided that $P_{peak} > \bar{P}$. If this does not hold, it is easy to show that the optimal power policy satisfying the peak transmit power constraint as well as the coupling constraint is given as $P^{\star}_{s,k} = P_{peak}$.

by solving, with respect to z, the equation

$$\beta_k z \exp(z) E_1(z) - \frac{\lambda \pi_k \bar{w} z}{g} + \alpha_k = 0. \quad (23)$$

While finding a closed form solution for equation (23) is not possible, one can produce accurate approximations for this solution by employing the procedure described in the following analysis and discriminating cases regarding the value of the argument $\frac{\lambda \pi_k}{g}$.

A. Approximating the solution of (21) for $\frac{\lambda \pi_k}{g} \leq \mathcal{U}(\bar{w} - N_{0,r}; \alpha, \beta)$

In this case, due to monotonicity of $\mathcal{U}(x; \alpha, \beta)$, it is easy to see that the solution x of (21) satisfies the condition $x \geq \bar{w} - N_{0,r}$. Thus, the solution z of (23) is such that $z \geq 1$. One can then approximate the solution of equation (23) by exploiting the following approximation for function $z \exp(z) E_1(z)$, [17, eq. 5.1.54]

$$z \exp(z) E_1(z) \approx \frac{z^2 + a_1 z + a_2}{z^2 + b_1 z + b_2}, \text{for} \quad z \geq 1, \quad (24)$$

with coefficients a_1, a_2, b_1 and b_2 defined in [17, eq. 5.1.54]. By employing (24) in (23), one can approximate the solution of (23) by finding the solution of the cubic equation

$$c_1 z^3 + c_2 z^2 + c_3 z + c_4 = 0, \quad (25)$$

where coefficients $c_1, \ldots, c_4$ can be easily found by substituting (24) in (23), provided that such a solution can be found that satisfies the constraint $z \geq 1$. If such a solution cannot be found, one can select to set $z = 1$.

B. Approximating the solution of (21) for $\frac{\lambda \pi_k}{g} \geq \mathcal{U}(\bar{w} - N_{0,r}; \alpha, \beta)$

In this case, given also the positivity constraint for variable x, it is easy to show that the solution of equation (23) should belong in the interval $z \in \left[\min\left\{\frac{N_{0,r}}{\bar{w}}, 1\right\}, 1\right]$. Thus one can find the optimal PP by applying any iterative root finding method in this closed interval. Having derived the optimal PP and developed an approximation technique for its calculation, in the following section we exploit these results within the context of joint SS and PP optimization.

VI. Joint SS and PP optimization

We are interested in the problem of joint SS and PP optimization such as to maximize the STx-SRx average communication rate, subject to PTx-PRx communication QoS constraints. In more detail, emphasizing on the reliable detection of the presence of a PTx, we consider that the CR system operates given a predefined detection probability constraint $\mathcal{P}_d = \tilde{\mathcal{P}}_d$ in addition to the constraints of problem (12). Based on the abovementioned constraints, the optimal joint PP and SS optimization problem is expressed as

$$\begin{aligned} &\underset{P_{s,k}(g), k\in\{0,1\}, \varepsilon, N}{\text{maximize}} : \ \mathcal{C} = \mathcal{C}_0 + \mathcal{C}_1 \\ &\text{subject to:} \quad (1-\mathcal{P}_d) E_g\{P_{s,0}(g)\} + \mathcal{P}_d E_g\{P_{s,1}(g)\} \leq \bar{P}, \\ &0 \leq P_{s,0}(g), P_{s,1}(g) \leq P_{peak}, \text{and} \quad \mathcal{P}_d = \tilde{\mathcal{P}}_d. \end{aligned} \quad (26)$$

Finding an exact solution to problem (26) appears to be cumbersome mainly due to complexity of expressions for $\mathcal{P}_d$. As a result, an alternative approach is needed in order to address the problem of joint SS and PP optimization. Such an approach is presented in what follows, where an alternating maximization iterative algorithm is proposed for the joint SS and PP optimization. This algorithm is further described in Algorithm 1.

Algorithm 1 Joint SS and PP optimization given PTx-PRx QoS constraints

1 Initialization ($m = 0$). Randomly select a pair of sensing parameters $\left(N^{(0)}, \varepsilon^{(0)}\right), N^{(0)} \leq T f_s$, satisfying the constraint $\mathcal{P}_d\left(N^{(0)}, \varepsilon^{(0)}\right) = \tilde{\mathcal{P}}_d$ and set $m = m + 1$.
2 For the m-th iteration, given $\left(N^{(m-1)}, \varepsilon^{(m-1)}\right)$, calculate the optimal PP $P_{s,k}^{(m)}(g), k \in \{0,1\}$ by solving the optimization problem (12).
3 Given $P_{s,k}^{(m)}(g), k \in \{0,1\}$, apply an exhaustive search procedure to allocate the optimal pair $\left(N^{(m)}, \varepsilon^{(m)}\right), N^{(m)} \leq T f_s$ satisfying the constraint $\mathcal{P}_d\left(N^{(m)}, \varepsilon^{(m)}\right) = \tilde{\mathcal{P}}_d$, that maximizes the rate $\mathcal{C}$ in (12).
4 Set $m = m + 1$. If $m \leq m_{\max}$ go to Step 2. Otherwise set $(N^\star, \varepsilon^\star) = \left(N^{(m-1)}, \varepsilon^{(m-1)}\right)$ and continue to Step 5.
5 Given $(N^\star, \varepsilon^\star)$ calculate the final PP $P_{s,k}^\star(g), k \in \{0,1\}$ by solving (12).

In the following section, we examine, by means of simulation, the achievable performance when the predescribed iterative SS and PP optimization algorithm is used.

VII. Numerical Results

In what follows Monte Carlo results are presented that depict the performance achieved by the presented hybrid CR system model when the joint SS and PP optimization algorithm presented in Algorithm 1 is applied. A CR system is examined that is characterized by a MAC frame of size equal to $100 msec$. The sampling frequency f_s for SS is selected to be $f_s = 6MHz$. Regarding the noise characteristics of STx and SRx, for the sake of simplicity, we assume that $N_{0,t} = N_{0,r} = 0[dB]$. A relatively reliable PTx-STx channel is assumed with an average SNR $\bar{\gamma}_t = 10[dB]$ such that reliable SS can be achieved. A high target detection probability $\tilde{\mathcal{P}}_d = 0.9$ is considered. Moreover, we assume that secondary communication is mostly interference limited, i.e., we set $\bar{w} = 5[dB]$. Regarding fading conditions, we assume that $\sigma_{ss}^2 = 0[dB]$ and $\sigma_{sp}^2 = -3[dB]$. Considering primary user activity, two different scenarios are examined. Initially a scenario with high primary activity is investigated, by setting $\mathcal{P}_1 = 0.7$. In Fig. 1 the achievable ergodic rate of secondary communication for the derived joint SS and PP optimization algorithm is plotted as a function of the average interference constraint $\mathcal{I}$ for the predescribed system. The application of the exact PP as derived in Section IV as well as the application of the approximation method presented in Section V are considered. For the simulations shown in Fig. 1, the peak power constraint P_{peak} is selected such that $P_{peak}/N_{0,r} = 20[dB]$. For the sake of comparison, in the same figure the performance of conventional interweave and underlay systems is also shown.

Concerning the investigated conventional underlay system, a PP maximizing the ergodic rate of the secondary system under the same average interference and peak transmit power constraints, considering the same CSIT availability is applied. Working in a similar manner as in the investigated hybrid system, it can be shown that this policy can be derived by solving the following optimization problem

$$\begin{aligned}&\underset{P_{und}(g)}{\text{maximize}:}\\&E_g\left\{\log_2\left(1+\frac{gP_{und}(g)}{N_{0,r}}\right)+\frac{\mathcal{P}_1}{\ln 2}\left(U\left(gP_{und}(g)\right)-U(0)\right)\right\}\\&\text{subject to:}\quad E_g\left\{P_{und}(g)\right\}\le\bar{P},0\le P_{und}(g)\le P_{peak}.\end{aligned} \tag{27}$$

One can show that this problem can be solved by applying an optimization procedure similar to the one presented in Section IV that is not presented here due to space limitations. Note that the underlay CR PP that is derived by this procedure is also a novel result, since in the past, no underlay CR PP has been presented that takes into account the primary user activity profile.

On the other hand, for the investigated interweave system a joint SS and PP optimization procedure is applied that is similar to the one presented in Algorithm 1. That is, we iteratively optimize the SS time and PP design such as to maximize the achievable rate for the interweave system, subject to the same average interference constraint, peak transmit power constraint and target detection probability $\tilde{\mathcal{P}}_d$. Regarding the PP optimization problem for the interweave system, it is easy to show that it can be expressed as

$$\begin{aligned}&\underset{P_{int}(g)}{\text{maximize}:}\\&E_g\left\{(\alpha_0+\beta_0)\log_2\left(1+\frac{gP_{int}(g)}{N_{0,r}}\right)\right.\\&\left.+\frac{\beta_0\left(U\left(gP_{int}(g)\right)-U(0)\right)}{\ln 2}\right\}\\&\text{subject to:}\\&(1-\mathcal{P}_d)\,E_g\left\{P_{int}(g)\right\}\le\bar{P},0\le P_{int}(g)\le P_{peak},\\&\text{and}\ \ \mathcal{P}_d=\tilde{\mathcal{P}}_d.\end{aligned} \tag{28}$$

As in the case of interweave CR, optimization problem (28) can also be solved applying a procedure similar to the one developed in Section IV that is not shown here due to space limitations.

By observing the results presented in Fig. 1, it is easy to see that the approximation procedure for the optimal PP presented in Section V results in very similar performance as the actual optimal PP presented in Section IV. This is confirmed by the fact that the average achievable secondary rates for the two PPs almost coincide. Moreover, it is easy to see that for small values of the average interference constraint $\mathcal{I}$, the interweave CR system achieves higher rates than the underlay one. This trend is reversed as $\mathcal{I}$ increases. One can also note the fact that for high values of the average interference constraint $\mathcal{I}$, the achievable secondary rate of the interweave CR system, is observed to saturate. This is due to the fact that for high values of $\mathcal{I}$ and specifically for values of $\mathcal{I}$ that lead to $\bar{P}\ge P_{peak}$, it is easy to show that the optimal interweave PP becomes the fixed PP $P_{int}=P_{peak}$. A similar effect, i.e., the reduction

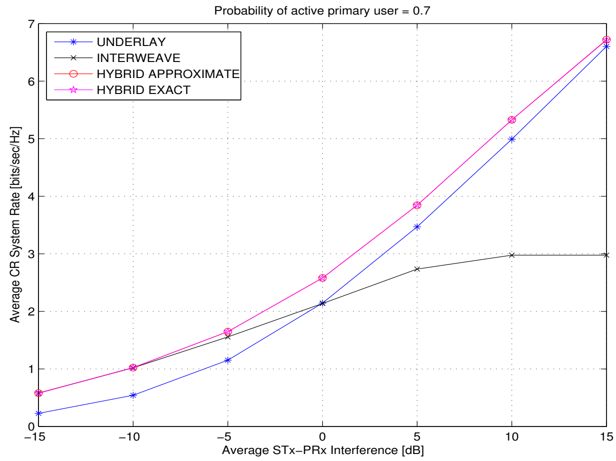

Fig. 1. The average rate of the investigated hybrid CR system, as a function of the average STx-PRx interference constraint $\mathcal{I}$. For the sake of comparison the performance of the two conventional CR systems, i.e., interweave and underlay is also plotted. A scenario with high primary activity, i.e., ($\mathcal{P}_1=0.7$) is considered.

of the optimal PP to the fixed transmit PP, holds also for the underlay as well as the hybrid CR systems. Nonetheless, for the investigated system parameters this appears for high, impractical values of $\mathcal{I}$. Finally, we can see that the proposed joint SS and PP optimization framework, applied to the hybrid CR system, leads to performance benefits with respect to both interweave and underlay CR systems, particularly for intermediate values of the average interference constraint $\mathcal{I}$.

Regarding the calculation of the applied PPs for the underlay and interweave secondary systems, this has been done using the approximation method developed in Section V. This is due to the fact that by the application of the approximation developed in Section V for the calculation of the optimal PP for the hybrid CR system, it has become evident that this approximation achieves very similar results with the actual PP obtained numerically as described in Section IV. In addition, we should mention that in order to decrease the computational complexity associated with the exhaustive search for optimizing the sensing time for the interweave and hybrid CR systems, we have reduced the search space for the optimal sensing time to the set $\mathcal{S}=\left\{\frac{iT}{100}|i=0,\ldots,100\right\}$.

Finally, in Fig. 2 the performance of the hybrid CR system is investigated where this time, the case of a lower primary user activity profile is considered. In more detail, we assume that $\mathcal{P}_1=0.3$. Regarding the remaining system parameters we select them to be the same as in the first simulation scenario. Again, the fact that the approximation for the optimal PP presented in Section V achieves performance results very similar with the ones obtained by the actual optimal PP derived in Section IV can be observed. Based on the results shown in Fig. 2 it becomes clear, in a more emphatic manner than in Fig. 1, that for strict average STx-PRx interference constraints $\mathcal{I}$, interweave CR systems outperform underlay CR systems. Moreover, it is also evident that the proposed optimal SS and PP optimization framework outperforms, in terms of achievable rate, both interweave and underlay CR approaches. One can also notice that the achievable secondary rate for the interweave CR system saturates when $\mathcal{I}>10dB$.

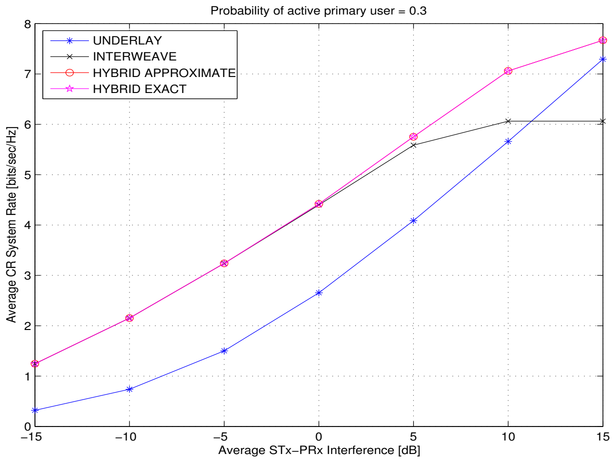

Fig. 2. The average rate of the investigated hybrid CR system, as a function of the interference constraint $\mathcal{I}$. For the sake of comparison the performance of the two conventional CR systems, i.e., interweave and underlay is also plotted. A scenario with low primary activity, i.e., $(\mathcal{P}_1 = 0.3)$ is considered.

Comparing the results presented in Figs. 1 and 2, it becomes evident that lower activity of primary user results in higher average rate for the secondary system. More importantly, one can notice that for strict values for the average interference constraint $\mathcal{I}$, the performance of the proposed hybrid system almost coincides with the performance of the interweave CR system, while for high values of $\mathcal{I}$ the performance of the hybrid system tends to coincide with the one of the underlay CR system.

VIII. Conclusion

A novel PP for optimizing the achievable rate of secondary hybrid CR systems based on the combination of underlay and interweave techniques has been presented in the existence of limited CSIT, and an algorithm for approximating this PP has been developed. By exploiting the derived PP, a novel iterative algorithm for jointly optimizing SS and the applied PP for hybrid CR systems has been proposed. By means of simulations, it has been confirmed that the proposed algorithm results in increased achievable rates, as compared to conventional underlay and interweave CR systems subject to the same constraints. Finally, it has been observed that for strict constraints concerning the interference caused by the secondary system to primary communication, the use of the proposed optimization algorithm for the examined hybrid CR system, results in performance similar to that of the interweave CR system, while for looser interference constraints, the use of the proposed optimization algorithm results in performance results similar to the ones achieved by the underlay CR.

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